

## ANALYSIS AND DESIGN OF A FOLDED BRANCH LINE COUPLER

J P Starski, H B Lundén  
 Division of Network Theory  
 Chalmers University of Technology  
 Gothenburg, Sweden

V K Tripathi  
 Dept of Electrical and Computer Eng  
 Oregon State University  
 Corvallis, OR, USA

ABSTRACT

The scattering parameters of branch line hybrids with coupled lines are derived in terms of the even- and odd-mode admittances and the lengths of the lines. These are used to formulate closed form design formulas. The result gives very compact structures at lower frequencies. At higher frequencies the coupling between the lines due to the limited spacing can be compensated.

INTRODUCTION

Branch line hybrids [1] are used extensively at microwave frequencies for a host of applications. The major problems arising in the design of such hybrids are the junction effects, coupling between the lines, and for microstrip realizations dispersion effects. In addition, single section hybrids are essentially narrow band structures. Most of these problems have been studied by a number of workers [2]-[8]. However, analytical methods to incorporate the effect of coupling between the lines are not available. In this article the analysis of a folded branch line hybrid with coupled lines shown in Fig 1, are presented.

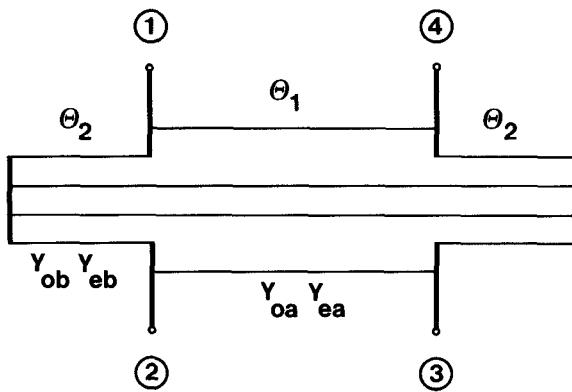


Fig 1 Schematic of a folded branch line coupler

COUPLER ANALYSIS AND DESIGN

The admittance parameters of the four-port can be found from the following equations [1]:

$$\begin{aligned} Y_{11} &= \frac{1}{2} (Y_{11e} + Y_{11o}) \\ Y_{12} &= \frac{1}{2} (Y_{11e} - Y_{11o}) \\ Y_{13} &= \frac{1}{2} (Y_{14e} - Y_{14o}) \\ Y_{14} &= \frac{1}{2} (Y_{14e} + Y_{14o}) \end{aligned} \quad (1)$$

where the even- and odd-mode admittances are given in Table 1.

TABLE 1

	Even-mode	Odd-mode
$Y_{11} = Y_{44}$	$\frac{1}{2} \left( \frac{Y_{ea}}{Y_{eb}} \right)^2 + \frac{1}{2} \left( \frac{Y_{eb}}{Y_{ea}} \right)^2$	$\frac{1}{2} \left( \frac{Y_{oa}}{Y_{ob}} \right)^2 + \frac{1}{2} \left( \frac{Y_{ob}}{Y_{oa}} \right)^2$
$Y_{14} = Y_{41}$	$\frac{1}{2} \left( \frac{Y_{ea}}{Y_{eb}} \right)^2 - \frac{1}{2} \left( \frac{Y_{eb}}{Y_{ea}} \right)^2$	$\frac{1}{2} \left( \frac{Y_{oa}}{Y_{ob}} \right)^2 - \frac{1}{2} \left( \frac{Y_{ob}}{Y_{oa}} \right)^2$

The scattering parameters of the four-port in terms of the even- and odd-mode excitations are given by:

$$\begin{aligned} S_{11} &= \frac{1}{2} (\Gamma_e + \Gamma_o) \\ S_{12} &= \frac{1}{2} (\Gamma_e - \Gamma_o) \\ S_{13} &= \frac{1}{2} (\Gamma_e - \Gamma_o) \\ S_{14} &= \frac{1}{2} (\Gamma_e + \Gamma_o) \end{aligned} \quad (2)$$

Under the assumption that the symmetrical four-port is terminated in the conductance  $G$  at all ports, the even- and odd-mode reflection and transmission coefficients are

$$\begin{aligned} \Gamma_{e,o} &= \frac{G^2 - Y_{11e,o}^2 + Y_{14e,o}^2}{(G + Y_{11e,o})^2 - Y_{14e,o}^2} \\ T_{e,o} &= \frac{-2Y_{14e,o} - G}{(G + Y_{11e,o})^2 - Y_{14e,o}^2} \end{aligned} \quad (3)$$

The design, of the hybrid shown in Fig 1, is based on the criterion that all the four ports are matched and that the division of power between the coupled and direct port is in accordance with the specification at the frequency of interest. Thus we have

$$G^2 = Y_{11e}^2 - Y_{14e}^2 = Y_{11o}^2 - Y_{14o}^2 \quad (4a)$$

and

$$\frac{S_{14}}{S_{13}} = \frac{G(Y_{14e} + Y_{14o}) + Y_{11e}Y_{14o} + Y_{11o}Y_{14e}}{G(Y_{14o} - Y_{14e}) + Y_{11e}Y_{14o} - Y_{14o}Y_{11e}} \quad (4b)$$

The derivation of the compensation equations is made in two steps

1. Consider the folded branch lines as uncoupled.

Thus, the matching conditions are given by:

$$Y_{ea}^2 - Y_b^2 \tan^2 \theta_2 + 2Y_b Y_{ea} \tan \theta_2 \cot \theta_1 = 1 \quad (5a)$$

$$Y_{oa}^2 - Y_b^2 \cot^2 \theta_2 - 2Y_b Y_{oa} \cot \theta_2 \cot \theta_1 = 1 \quad (5b)$$

where

$$Y_b^2 = Y_{ob} \cdot Y_{eb}$$

Then, for a 3 dB hybrid, the power division eq (4b) gives

$$Y_b^2 - Y_{ea} Y_{oa} \cot^2 \theta_1 + Y_b \cot \theta_1 (Y_{oa} \tan \theta_2 - Y_{ea} \cot \theta_2) = 1 \quad (5c)$$

The set of equations (5) is undetermined since there are five variables and only three equations. Hence there is an unlimited number of solutions to the equation set (5). However, only some of them are physically realizable.

If  $\theta_2$  is chosen to be  $\pi/4$ , as in the case of an ordinary branch line coupler, the solution of the matching eqs (5a-b) is given by:

$$\theta_1 = \operatorname{arccot} \left[ \frac{Y_{oa} - Y_{ea}}{2Y_b} \right] \quad (6)$$

Eq (6) is independent of the power division, eq (5c).

2. Consider the low impedance lines as uncoupled and the folded branch lines as coupled, e.g.  $Y_{ea} = Y_{oa}$  and  $Y_{eb} \neq Y_{ob}$ .

The matching conditions for this case are given by:

$$Y_a^2 \csc^2 \theta_1 - (Y_a \cot \theta_1 - Y_{eb} \tan \theta_2)^2 = 1 \quad (7a)$$

$$Y_a^2 \csc^2 \theta_1 - (Y_a \cot \theta_1 + Y_{ob} \cot \theta_2)^2 = 1 \quad (7b)$$

where

$$Y_a^2 = Y_{oa} \cdot Y_{ea}$$

As in the previous case the eqs (7a-b) are undetermined. However, when choosing  $\theta_1$  to be  $\pi/2$  the solution of eqs (7a-b) is given by:

$$\theta_2 = \operatorname{arctan} \sqrt{\frac{Y_{ob}}{Y_{eb}}} \quad (8)$$

The derived compensation equations for the folded branch line hybrid in Fig 1 are then given by eqs (6) and (8). The admittances of the uncoupled hybrid,

$Y_a = \sqrt{Y_{oa} Y_{ea}}$  and  $Y_b = \sqrt{Y_{ob} Y_{eb}}$ , are related to the

desired power division by eq (4b).

The eqs (6) and (8) show that the compensation of the length of the low impedance line is only due to the coupling between the low impedance lines, e.g. it is possible to compensate for coupling in an ordinary branch line coupler where only the main lines are coupled.

The theoretical frequency response of a 3 dB branch line coupler with different coupling coefficients between the lines is shown in Figs 2 (a-d). Figs 3 (a-d) show the measured results of a 2.4 dB coupler. The coupling between the lines was 20 dB. Figs 4 (a-d) show the theoretical frequency response of the measured hybrid. As can be observed the measurements show a very good agreement with the theory.

#### REFERENCES

- [1] J Reed and G J Wheeler, "A method of analysis of symmetrical four-port networks", IRE Trans Microwave Theory Tech, Vol MTT-44, pp 246-256, Oct 1956.
- [2] R Levy and L F Lind, "Synthesis of symmetrical branch-guide directional couplers", IEEE Trans MTT, pp 80-89, Feb 1968.
- [3] G P Riblet, "A directional coupler with very flat coupling", IEEE Trans Microwave Theory Tech, Vol MTT-26, pp 70-74, Feb 1978.
- [4] T Okoshi et al, "Computer oriented synthesis of optimum circuit pattern of 3 dB hybrid ring by planar circuit approach", IEEE Trans Microwave Theory Tech, Vol MTT-29, pp 194-202, March 1981.
- [5] G L Matthaei, L Young and E M T Jones, "Microwave filters impedance matching networks, and coupling structures", New York: McGraw-Hill, 1964.
- [6] E O Hammerstad and F Bekkadal, "Microstrip Handbook", ELAB Report STF 44 A74169, The University of Trondheim, The Norwegian Institute of Technology, 1975.
- [7] W H Leighton and A G Milnes, "Junction reactance and dimensional tolerance effects on X-band 3 dB directional couplers", IEEE Trans Microwave Theory Tech, Vol MTT-19, pp 818-824, Oct 1971.
- [8] M Dydyk, "Master the T-junction and sharpen your MIC designs", Microwaves, Vol 16, pp 184-186, May 1977.

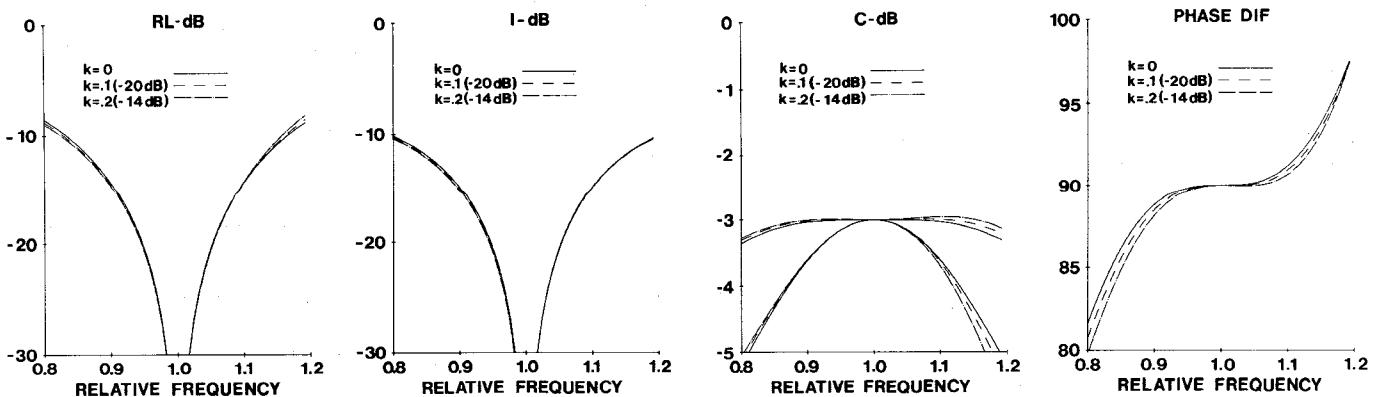


Fig 2 Computer generated frequency response of a 3 dB folded branch line coupler with different coupling coefficients between the lines.

a) Return loss, b) Isolation, c) Coupling, d) Phase difference between the direct and the coupled port

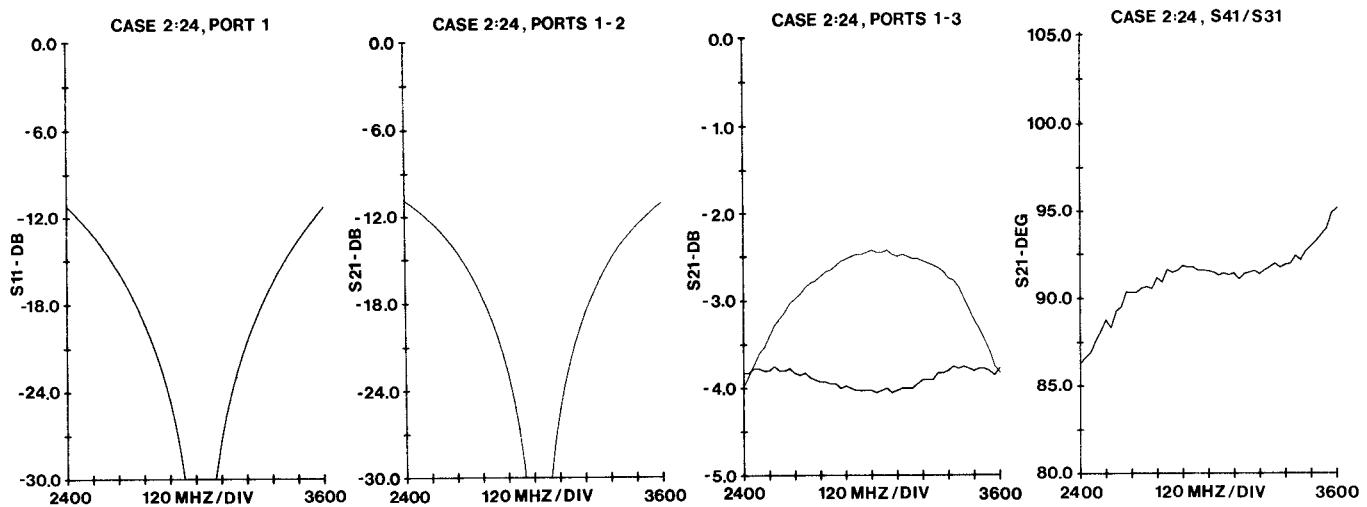


Fig 3 Measured performance of a 2.4 dB folded branch line coupler  
 a) Return loss, b) Isolation, c) Coupling, d) Phase difference between the direct and the coupled port

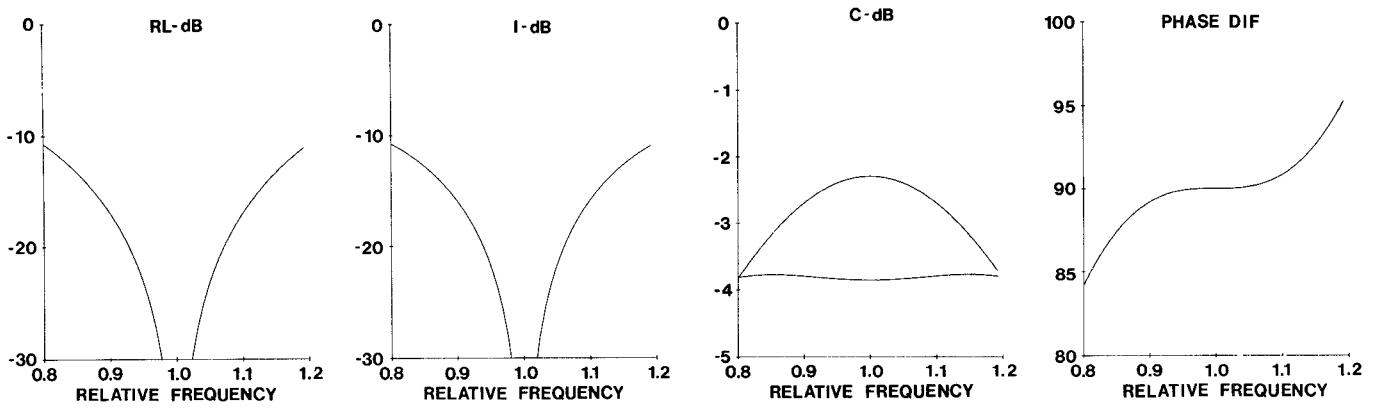


Fig 4 Computer generated frequency response of a 2.4 dB folded branch line coupler  
 a) Return loss, b) Isolation, c) Coupling, d) Phase difference between the direct and the coupled port